Solihull
METROPOLITAN
BOROUGH COUNCIL

USING RESOURCES EFFECTIVELY TO TEACH

Compiled by the 'Solihull LA Primary Mathematics Forum' 2019

Using resources effectively to teach fractions

Through consultation with schools, and question level analysis of 2017 KS2 SAT papers, the primary maths forum identified the need for support with the teaching of fractions. This document offers ideas to teachers demonstrating how to teach fractions through a CPA approach (concrete, pictorial, abstract) progressively from year 1 through to year 6. The fractions domain has been divided into separate key areas which include

- Counting in fractions
- Recognising and identifying fractions
- Comparing and ordering fractions (incl. equivalence)
- Adding and subtracting fractions
- Multiplying and dividing fractions

For each key area, a variety of ideas are presented illustrating how concrete resources/ manipulatives can be used in teaching as well as pictures and images and abstract representations for each concept. Some links between the key areas are made explicit within the document, however teachers are encouraged to look at all areas within their year group (and previous year groups) to gain a wider understanding of progression and the links between concepts.

Some ideas have been inspired by activities found in:

- White Rose Hub materials
- Maths No Problem
- Classroom Secrets
- Collins Shanghai
- Twinkl

Primary Mathematics Forum Contributors: With thanks to:

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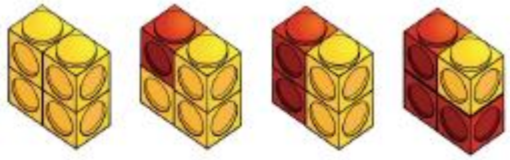
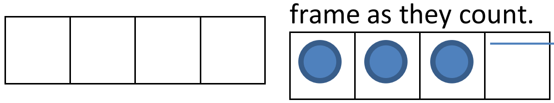
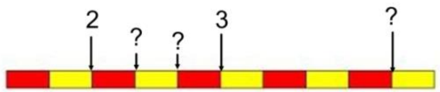
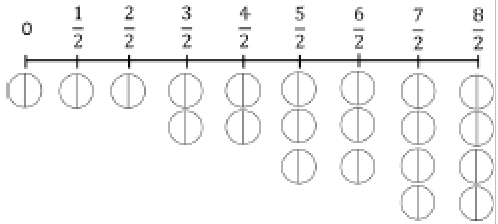
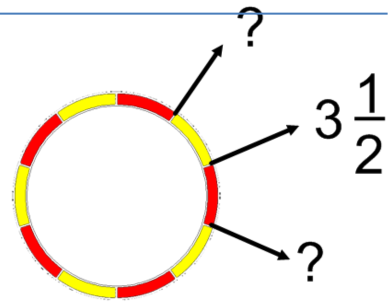
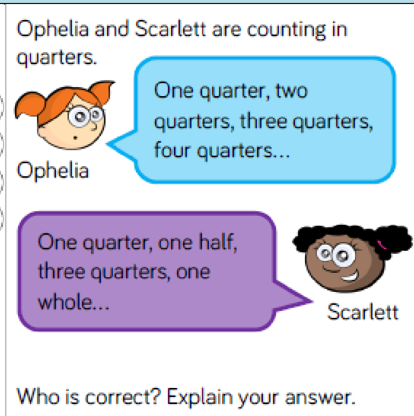
Using resources effectively to teach…. COUNTING IN FRACTIONS

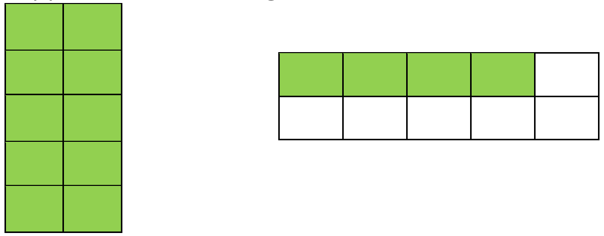
Obj	Teaching points	Concrete	Pictorial	Abstract
Year 2: Count in fractions up to ten, using ½ and $^2/_4$ equivalents.	• Children need to be able to count forwards and backwards from different starting points. • It is important that children have multiple opportunities to experience counting using C.P.A. • Before introducing equivalent fractions when counting, children need a secure understanding of these.	 White Rose Yr2 Sp 4 Using the picture below, give children fraction frames and allow them to place counters into the frame as they count.  Children might say: one quarter, two quarters (one half), three quarters, four quarters (one whole). Use the  counting stick, starting at different numbers as well as starting at 0. Practice counting forwards and backwards. In this example, children are expected to spot the pattern of counting in thirds. Then further apply this knowledge by finding the third missing number.	 White Rose Yr2 Sp 4 Use shading to count.  A counting hoop works in the same way as a counting stick. Here the discussion is about the pictorial representation, but it can be used as a concrete resource as well.	 White Rose Yr2 Sp 4 For every piece of ribbon, Sam chops it into 4 equal pieces. What fraction does each piece represent? If Sam has six pieces of ribbon, how many quarters does he have? Ext: How many halves does he have?

Year 3: Count up and down in tenths

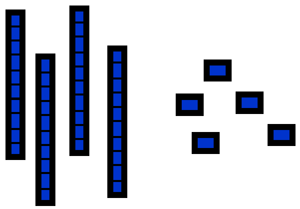
All of the Year 2 teaching points are applicable to this objective.

- Children need to understand that one tenth can be one whole divided into ten equal parts, but could be multiple wholes divided into ten equal parts.
- Children should be able to count forwards and backwards beyond one.

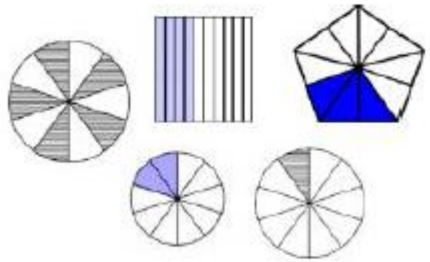
Children could use multiple tens frames to support them counting in tenths.



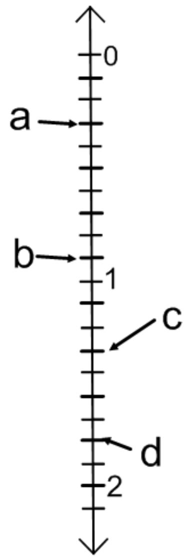
The children could use counters or shading. It is important that they can use the tens frame in different orientations.

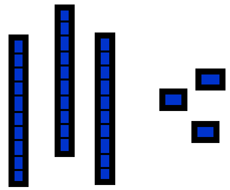


To start with, use the tenths cubes to embed the idea that 10 tenths = 1 whole. Children can then exchange their ten tenths for a 1 whole stick. Start at different points to rehearse counting forwards and backwards.



What does each image represent? How many more sections would I need to shade to make $2\frac{7}{10}$?





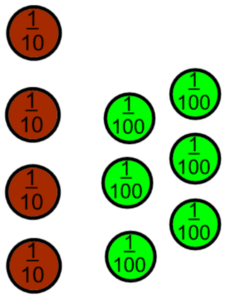
Sam started counting at $2\frac{7}{10}$. He finished with the picture above. How many tenths has he counted on?
Ext: This is Sam's finished image. Where could he have started counting? How many tenths away from this is it?

$2\,^{6}/_{10}$	$2\,^{7}/_{10}$			
	$3\,^{7}/_{10}$		$3\,^{9}/_{10}$	
		$4\,^{8}/_{10}$		

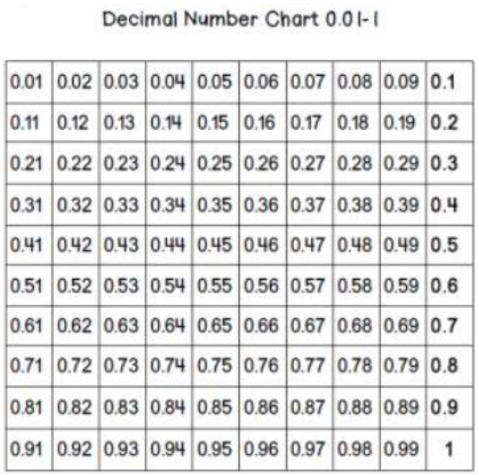
Here is part of a number square. Fill in the missing numbers.
(N.B. to extend more able learners, you may wish to remove $3\,^{9}/_{10}$ or extend the table beyond a whole.)

<table>
<tr>
<td>Year 4: Counting up and down in hundredths.</td>
<td>All of the previous teaching points are applicable to this objective.

Children need to understand that one hundredth is the same as dividing by 100.
Children need to see and be taught the equivalence between multiple hundredths and tenths e.g. $^{70}/_{100}$= $^{7}/_{10}$.

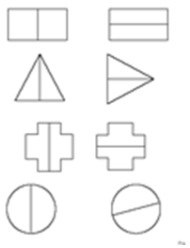
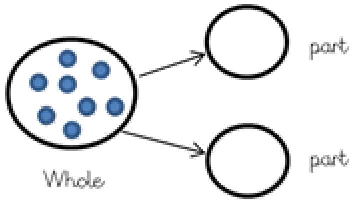
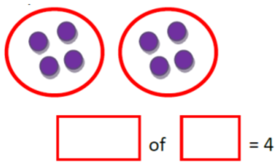
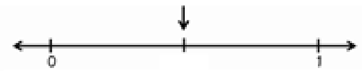
</td>
<td>Use 100 bead strings to rehearse counting forwards and backwards. Use different starting points and draw discussion into the fact that $^{1}/_{10}$ = $^{10}/_{100}$.
<hr>


To start with, use the hundredths counters to embed the idea that 10 hundredths = 1 tenth. Children can then exchange their ten hundredths for a tenth counter. Start at different points to rehearse counting forwards and backwards.</td>
<td>As for counting in tenths
Use dienes units blocks to represent 1/100
Use pennies to recognise 1/100 of a pound etc

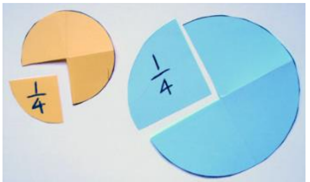
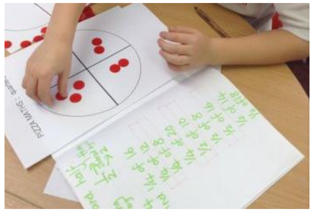
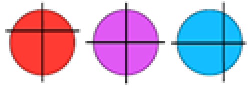
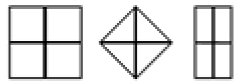
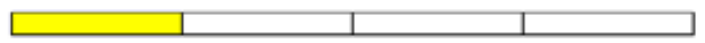
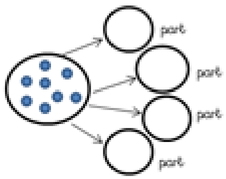
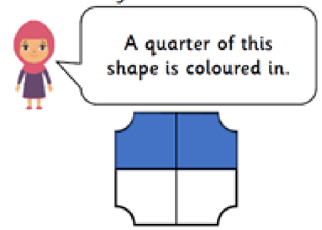
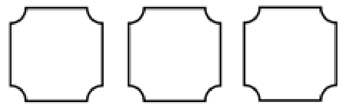

Decimal Number Chart 0.01-1
<table>
<tr><td>0.01</td><td>0.02</td><td>0.03</td><td>0.04</td><td>0.05</td><td>0.06</td><td>0.07</td><td>0.08</td><td>0.09</td><td>0.1</td></tr>
<tr><td>0.11</td><td>0.12</td><td>0.13</td><td>0.14</td><td>0.15</td><td>0.16</td><td>0.17</td><td>0.18</td><td>0.19</td><td>0.2</td></tr>
<tr><td>0.21</td><td>0.22</td><td>0.23</td><td>0.24</td><td>0.25</td><td>0.26</td><td>0.27</td><td>0.28</td><td>0.29</td><td>0.3</td></tr>
<tr><td>0.31</td><td>0.32</td><td>0.33</td><td>0.34</td><td>0.35</td><td>0.36</td><td>0.37</td><td>0.38</td><td>0.39</td><td>0.4</td></tr>
<tr><td>0.41</td><td>0.42</td><td>0.43</td><td>0.44</td><td>0.45</td><td>0.46</td><td>0.47</td><td>0.48</td><td>0.49</td><td>0.5</td></tr>
<tr><td>0.51</td><td>0.52</td><td>0.53</td><td>0.54</td><td>0.55</td><td>0.56</td><td>0.57</td><td>0.58</td><td>0.59</td><td>0.6</td></tr>
<tr><td>0.61</td><td>0.62</td><td>0.63</td><td>0.64</td><td>0.65</td><td>0.66</td><td>0.67</td><td>0.68</td><td>0.69</td><td>0.7</td></tr>
<tr><td>0.71</td><td>0.72</td><td>0.73</td><td>0.74</td><td>0.75</td><td>0.76</td><td>0.77</td><td>0.78</td><td>0.79</td><td>0.8</td></tr>
<tr><td>0.81</td><td>0.82</td><td>0.83</td><td>0.84</td><td>0.85</td><td>0.86</td><td>0.87</td><td>0.88</td><td>0.89</td><td>0.9</td></tr>
<tr><td>0.91</td><td>0.92</td><td>0.93</td><td>0.94</td><td>0.95</td><td>0.96</td><td>0.97</td><td>0.98</td><td>0.99</td><td>1</td></tr>
</table>
</td>
<td>As for counting in tenths
9.8 is equivalent to 980 hundredths,
9.82 is equivalent to ? hundredths.
How many hundredths are there in 10.0?

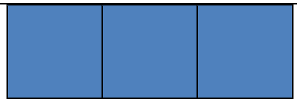
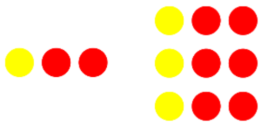
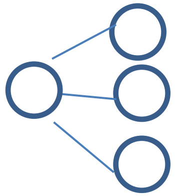
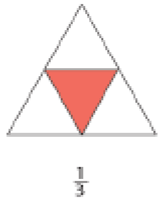
Using a partially filled in hundredths square, ask how you know where e.g. 2.76 would go.
<table>
<tr><td>2.52</td><td>2.53</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>2.67</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2.82</td><td></td><td></td><td>2.85</td><td>2.86</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>2.96</td><td></td></tr>
</table>
</td>
</tr>
</table>

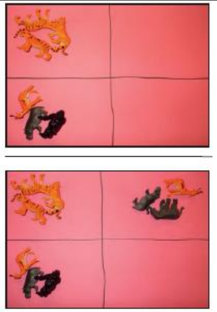
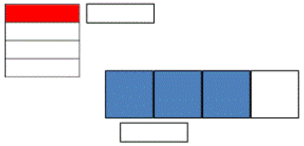
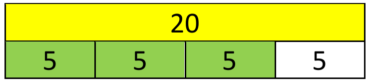
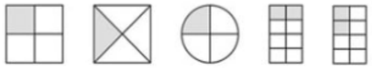
Using resources effectively to teach…. RECOGNISING FRACTIONS

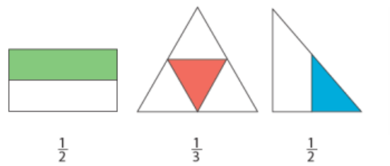
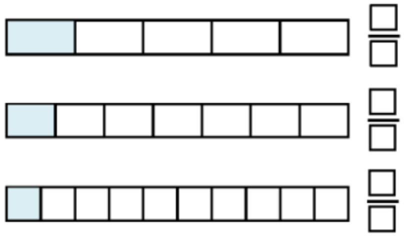
Obj	Teaching points	Concrete	Pictorial	Abstract
Year 1: Recognise, find and name a half as one of two equal parts of an object, shape and quantity)	Children explore finding a half for the first time using shapes, sets of objects and small quantities. They will use the vocabulary 'half' and 'whole'. Children will not be expected to use the fractional notation of $^1/_2$ within their working, but it may be useful to introduce both $^1/_2$ and $^2/_2$ as the fractional representation. It is important that they know that a half means one of two **equal** parts. Link to measurement Using a number line marked 0 to 1 to introduce the numerical value of a half.	Halve physical objects and shapes   Find half of these bears  Find half of a jug of water, half the mass of an object or half of a length.  Cont…	 Colour one half of each shape  Using a bar model show one half Find one half of the whole amount   	 Mark on the number line where halfway should be.  What is half of the amount shown?

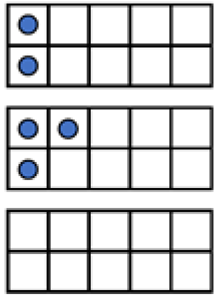
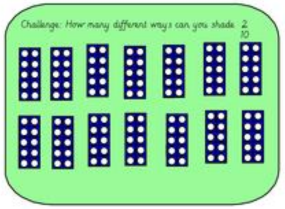
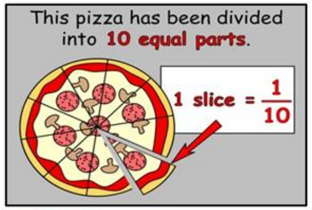
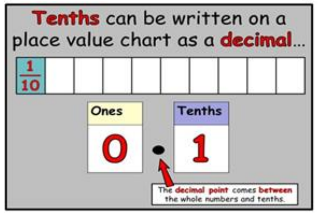
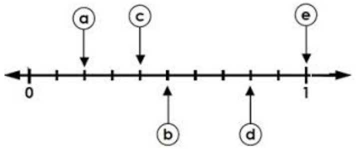
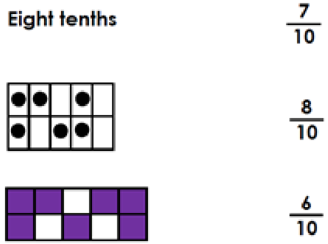
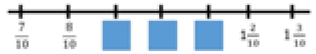
Year 1: Recognise, find and name a half as one of two equal parts of an object, shape and quantity)		…cont. Choose a number of counters. Place them onto 2 plates so that there is the same number on each half. When can you do this and when can't you? What do you notice? 		

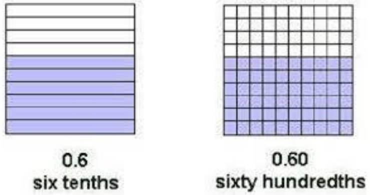
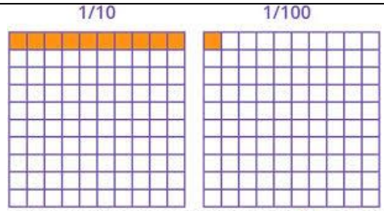
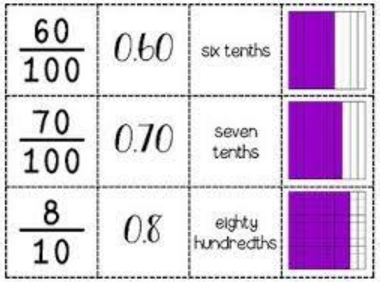
Year 1: Recognise, find and name a quarter as one of four equal parts of an object, shape and quantity)	Children explore finding a quarter for the first time using shapes, sets of objects and small quantities. They will use the vocabulary ‘quarter’ and ‘whole’. Children will not be expected to use the fractional notation of ¼ within their working, but it may be useful to introduce both $^{1}/_{4}$ and $^{4}/_{4}$ as the fractional representation. It is important that they know that a quarter means one of four **<u>equal</u>** parts. Link to measurement Using a number line marked 0 to 1 to introduce the numerical value of a quarter.	  	 Which shape has been split into quarters?  Shade one quarter of each shape  Find a quarter of objects  Represent a quarter on a bar model  Look at an amount as a whole and a quarter as one of four equal parts	6a. Aisha says:  Is she right? Explain your answer. True or false? Sharing 8 apples between 4 children means each child has 1 apple.  less than a quarter | exactly a quarter | more than a quarter Colour the shapes to match the labels

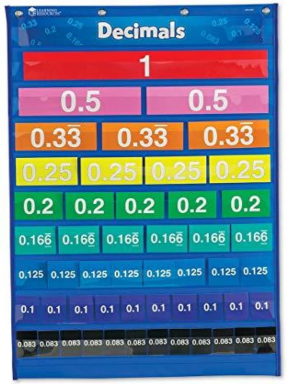
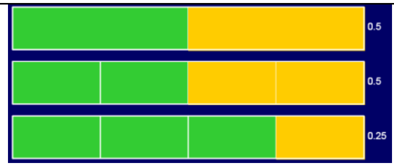
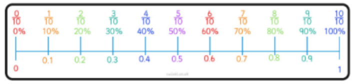
Year 2: Recognise, find, name and write one third ($^1/_3$) of length, shape, set of objects or quantity (Y2)	Children explore finding a third for the first time using shapes, sets of objects and small quantities. They will use the vocabulary 'third' and 'whole'. Children will not be expected to use the fractional notation of $^1/_3$ within their working, but it may be useful to introduce both $^1/_3$ and $^3/_3$ as the fractional representation. It is important that they know that a third means one of three **<u>equal</u>** parts. **(Y2)** Link to measurement Using a number line marked 0 to 1 to introduce the numerical value of a third.	 Strips of paper for children to find 1 out of 3 equal parts  Using counters to show the equal parts.  Fraction cards	Part whole diagrams to show 3 equal parts  9cm 3cm 3cm 3cm Bar model  Is this $^1/_3$ of this shape? Explain how you know.	Is $^1/_3$ of 9 greater than $^1/_2$ of 4? See similar questions to halves.

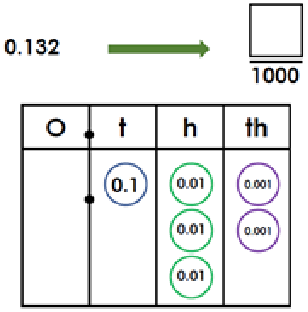
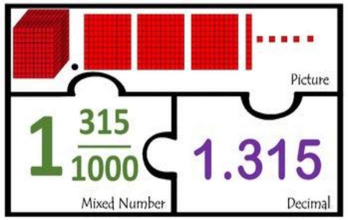
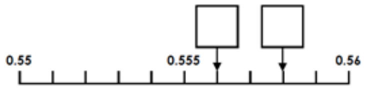
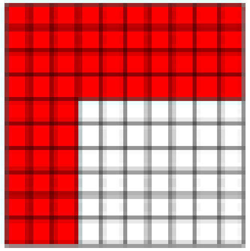
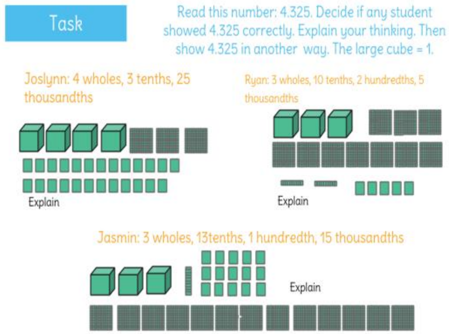
Year 2: Recognise, find, name and write quarters ($^1/_4$ $^2/_4$ $^3/_4$) of length, shape, set of objects or quantity	$^4/_4$ link to counting in fractions – the need to see wholes	 Practically investigating quarters. Folding paper to explore fractions 	 Name the fractions shown Children to use bar models to visually represent quarters? What is $^3/_4$ of 20?  Are all of these showing quarters? 	Is $^1/_4$ of 8 less than $^3/_4$ of 4?  Estimate the position of $^1/_3$, $^1/_4$ and $^3/_4$ $^1/_4$ of 4 = 1 $^1/_4$ of 8 = 2 $^1/_4$ of 12 = 3 Continue the pattern What do you notice? True or false? Half of 20cm = 5cm $^3/_4$ of 12cm = 9cm

Year 3: Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators	Identify fraction of shaded shape; position fractions on a number line; use fraction cards to show equivalence and compare fractions		 	 Continue the pattern. What do you notice? What about $^{1}/_{10}$ of 20? Use this to work out $^{2}/_{10}$ of 20, etc. True or false? $^{2}/_{10}$ of 20cm = 2cm $^{4}/_{10}$ of 40cm = 4cm $^{3}/_{5}$ of 20cm = 12cm

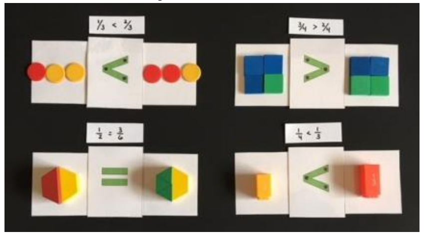
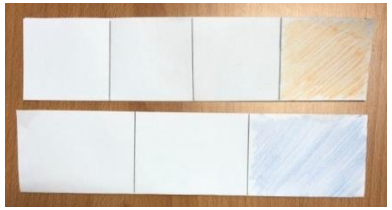
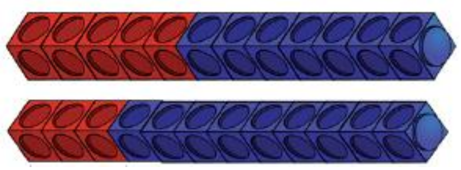
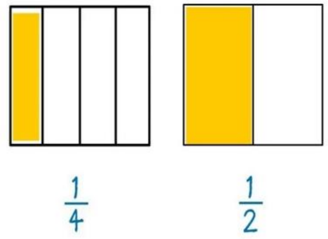
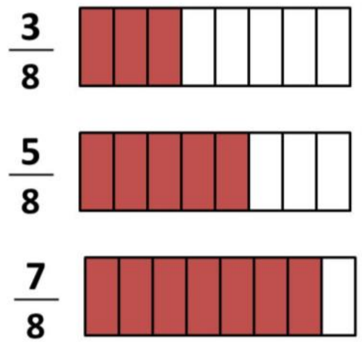
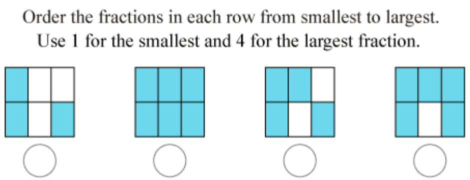
Year 3: Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10	Children explore finding a tenth using money (10p and pounds), sets of objects and small quantities. They will use the vocabulary 'tenth' and 'whole'. Children will use fractional representation for 1/10 etc. Link to money – the relationship between 10p and £1. Using a number line marked 0 to 1 to introduce the numerical value of a tenth. Dividing one-digit numbers by 10 to introduce the decimal notation.	 Use tens frames to represent tenths and count in tenths. Could also use a ten piece from numicon set with an object into the circles to represent the amount of tenths. Using 10p coins with 10 adding up to £1 - also links to the decimal place.  How many tenths of a whole pound do you have? 3/10 link to how it would be written as money £0.30.	    Which tenth is represented by the letter a? B? C? D? E? 	**What do you notice?** $^{1}/_{10}$ of 10 = 1 $^{2}/_{10}$ of 10 = 2 $^{3}/_{10}$ of 10 = 3 Continue the pattern. **What do you notice?** What about $^{1}/_{10}$ of 20? Use this to work out $^{2}/_{10}$ of 20, etc. $^{1}/_{10}$ of 100 = 10 $^{1}/_{100}$ of 100 = 1 $^{2}/_{10}$ of 100 = 20 $^{2}/_{100}$ of 100 = 2 **How can you use this to work out $^{6}/_{10}$ of 200? $^{6}/_{100}$ of 200?** 

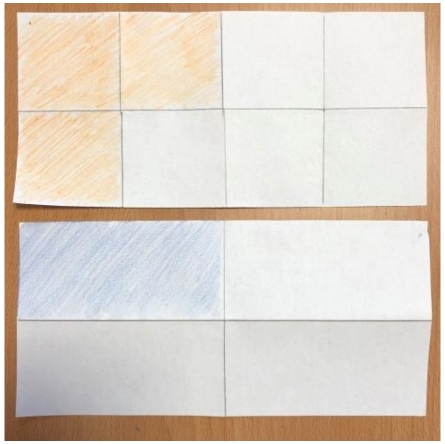
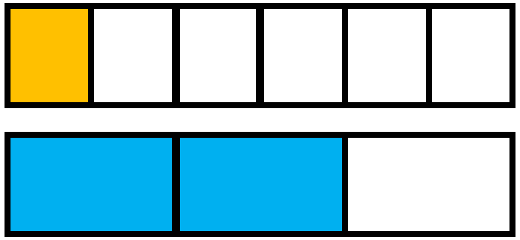
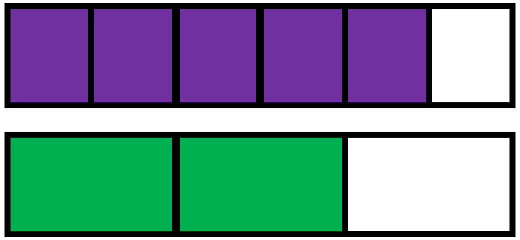
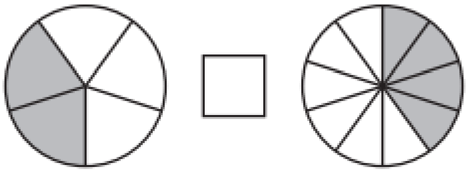
Year 4: Recognise and write decimal equivalents of any number of tenths or hundredths	Children explore finding any number of tenths or hundredths using money - 10p and 1p to pounds and write as the decimal equivalent.	  	 	**What do you notice?** One tenth of £41 One hundredth of £41 One thousandth of £41 Continue the pattern. What do you notice? 0.085 + 0.015 = 0.1 0.075 + 0.025 = 0.1 0.065 + 0.035 = 0.1 Continue the pattern for the next five number sentences

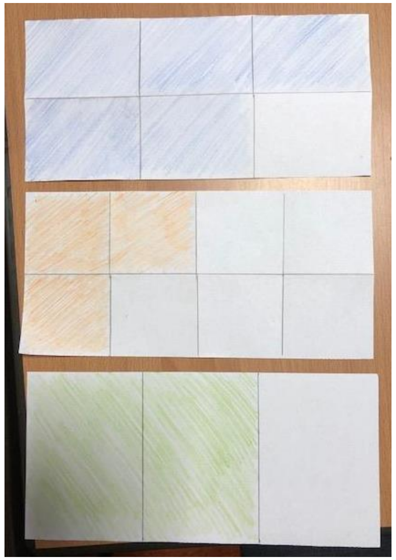
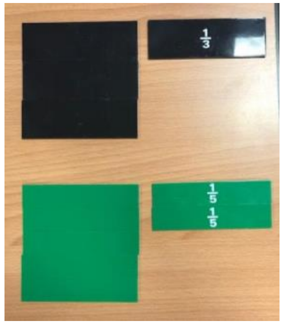
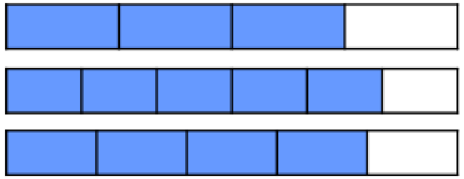
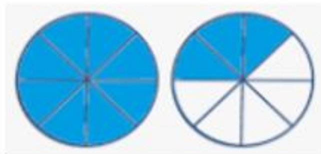
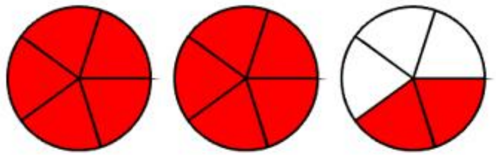
Year 4: Recognise and write decimal equivalents to $^{1}/_{4}$ $^{1}/_{2}$ $^{3}/_{4}$	Children revisit fraction equivalents and relate these images to decimals.	 	 Explore using link: https://mathsframe.co.uk/en/resources/resource/66/itp-fractions 	Match each fraction to its decimal equivalent. $\frac{1}{2}$ $\frac{4}{10}$ $\frac{3}{4}$ $\frac{1}{4}$ 0·25 0·75 0·4 0·5 Circle the equivalent fraction to 0·25. $\frac{2}{5}$ $\frac{5}{2}$ $\frac{25}{100}$ $\frac{100}{25}$ Round to the nearest whole number. $8\frac{3}{8}$ 8·38 8·83 Using these cards can you make a number between 4·1 and 4·61? 1 4 6 · What is the smallest number you can make using all four cards? What is the largest number you can make using all four cards?

Year 5: Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	Children to use place value counters to represent tenths and hundredths and relate these to decimals. Relate the teaching of thousandths to grams and ml in practical contexts.	 Use place value counters and grid to represent decimal numbers up to three decimal points and convert to fractions  Using base ten to physically represent decimal numbers.	 Use number lines to represent thousandths as the steps between hundredths.  Using a tens frame, 100 square, or thousands grid to represent tenths, hundredths and thousandths 	One tenth of £41 One hundredth of £41 One thousandth of £41 Continue the pattern What do you notice? 0.085 + 0.015 = 0.1 0.075 + 0.025 = 0.1 0.065 + 0.035 = 0.1 Continue the pattern for the next five number sentences. One thousandth of my money is 31p. How much do I have? True or false? 0.1 of a kilometre is 1m. 0.2 of 2 kilometres is 2m. 0.3 of 3 kilometres is 3m 0.25 of 3m is 500cm. $^{2}/_{5}$ of £2 is 20p True or false? 25% of 23km is longer than 0.2 of 20km. Convince me.

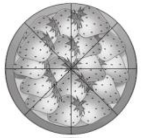
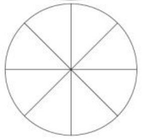
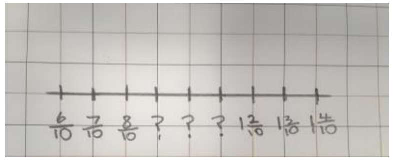
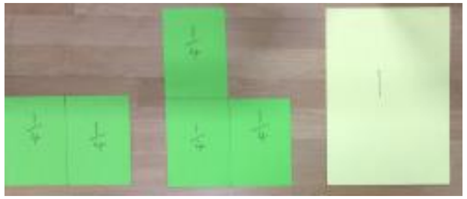
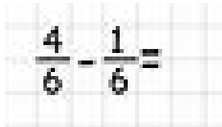
Using resources effectively to teach… COMPARING FRACTIONS

Obj	Teaching points	Concrete	Pictorial	Abstract
Year 3: compare and order unit fractions, and fractions with the same denominators	Unit fractions are fractions with a numerator that is 1. Language to be used is: compare order greater than and less than equal parts numerator denominator	Equipment that could be used: Paper strips; Counters; Cubes; Fraction tiles; Fraction rods; Cuisenaire rods E.g. Compare fractions using counters and other objects as shown below  Paper strips can be used to help compare fractions. Ensure that the paper strips are of equal size.   Use cube towers to compare fractions.	This could begin by using paper strips before exposing children to pictorial representations such as:    	For example: Compare using =, < or > $^1/_2$, $^1/_5$, $^1/_3$, $^3/_4$ Compare using =, < or > $^2/_7$, $^5/_7$, $^4/_8$, $^5/_8$ 1 2 3 4 4 $\frac{1}{4} < \frac{B}{C}$ Order the following fractions from smallest to largest:

Year 5: compare and order fractions whose denominators are all multiples of the same number	Language to be used is: compare order greater than and less than equal parts numerator denominator multiple Equivalent fractions need to be found so that all of the denominators are the same.	 Fraction tiles can be laid out to compare different sized fractions. 	 or  	For example: Order the following fractions from smallest to largest: $^2/_3$, $^3/_4$, $^2/_6$, $^1/_{12}$, $^5/_6$ Which is larger; $^5/_8$ or $^3/_4$ Using the number cards, can you complete the equation. Can you find more than one possibility? $\frac{7}{11} < \frac{\square}{\square} < \frac{8}{11}$ 11 15 22 25 33

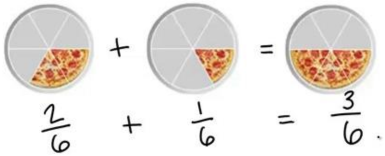
Year 6: compare and order fractions, including fractions >1	Language to be used is: Greater than Less than Equal Ascending Descending Numerator Denominator Equivalent fractions need to be found so that all of the denominators are the same.	  Building on to: Which is greater?	Order the fractions represented by the bar models in descending order:  Compare the fractions below. Explain which is larger and how you can prove it.  	Order the fractions below in descending order: $\frac{5}{6}$ $\frac{1}{4}$ $\frac{3}{10}$ $\frac{2}{5}$ $\frac{1}{2}$ Use < > or = to make the statement correct: $1\frac{2}{7} \bigcirc 1\frac{4}{5}$ $\frac{4}{\square} > \frac{3}{4}$ $\frac{\square}{4} < \frac{2}{5}$ $\frac{4}{\square} = \frac{1}{2}$ Can you use the digit cards to make the statement true using improper fractions 16 50 24 51 $\frac{\square}{\square} < \frac{\square}{\square}$

Using resources effectively to teach.... + AND - FRACTIONS

Obj	Teaching points	Concrete	Pictorial	Abstract
Year 3: Add and subtract fractions with the same denominator within one whole	To add and subtract fractions, the denominators must be the same.	  Provide pupils with a strawberry tart cut into eighths and an identically sized and cut blank copy. *Collins Shanghai Y3 Unit 8.4* Determine that each part represents one eighth of the tart because the whole has been divided into eight equal parts. Get the children to cut out each part of the pie and label them as $\frac{1}{8}$. Hold up one piece in each hand and elicit that this is $\frac{2}{8}$. Record the calculation: $\frac{1}{8}+\frac{1}{8}=\frac{2}{8}$. Relate the common denominators to the number of equal pieces of the tart, and then discuss how by adding two of them together they get $\frac{2}{8}$. Ask what would happen if one more eighth was added to the new strawberry tart. Stick another eighth on to get $\frac{3}{8}$. Continue this process. Put the final piece on and remind the children that $\frac{8}{8}$ is the same as one whole (strawberry tart). Cont…	Count up and down in fraction amounts on a number line.  *Twinkl* Count up in fraction amounts using paper cards.  *I See*  Cont…	Make sure the numerators are the same, then add the denominators. e.g. $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$ 

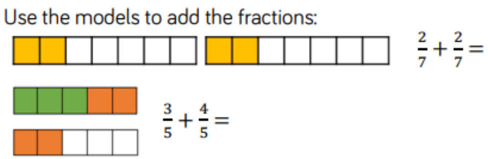
Year 3: Add and subtract fractions with the same denominator within one whole

...cont.

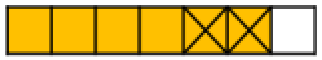


When using numicon, the base piece represents the denominator and the top pieces represent the numerator. Cubes or pegs could also be used to represent the numerators.

Cube towers can be used:

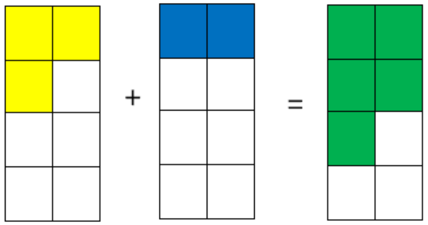


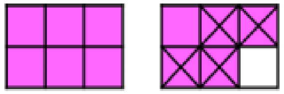
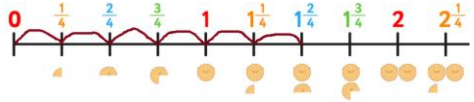
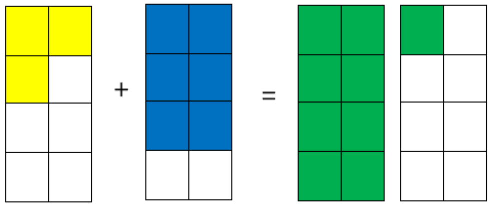
Similar resources can also be used to demonstrate subtraction of fractions.

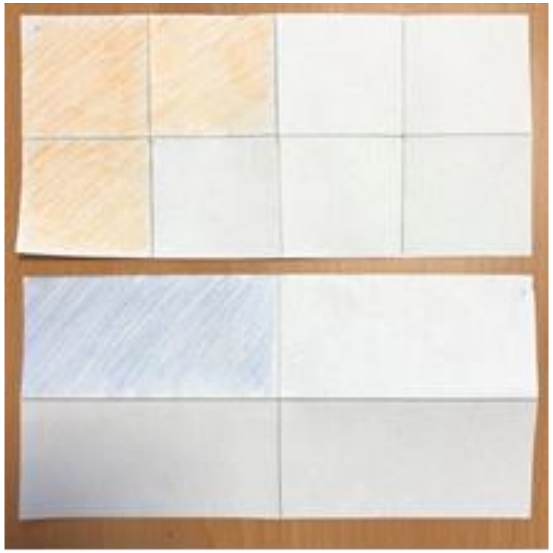
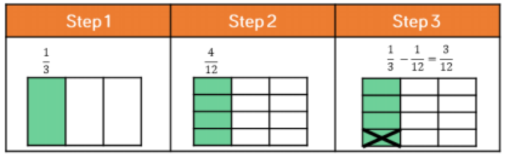
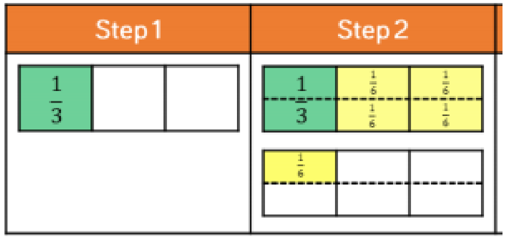


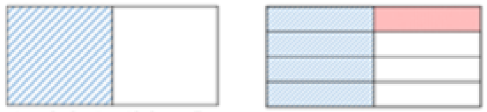
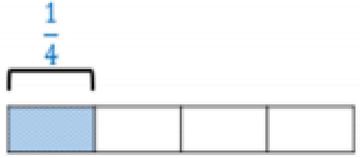
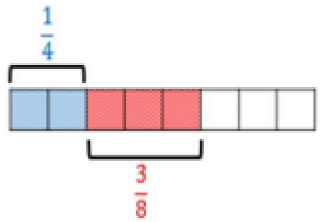
$\frac{6}{7} - \frac{2}{7} =$

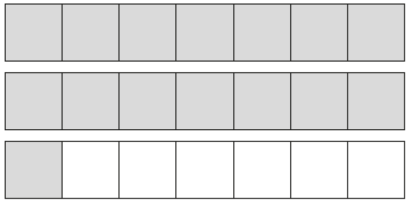
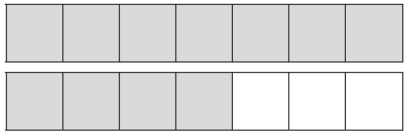
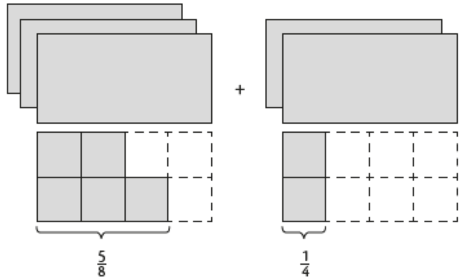
...cont.

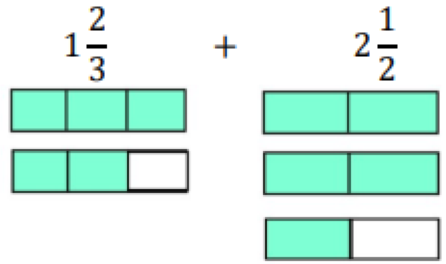
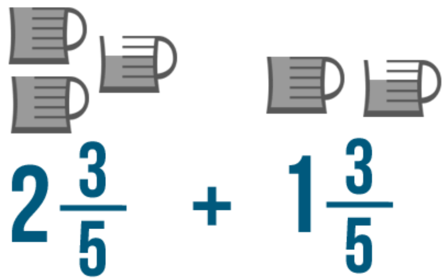
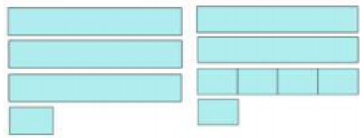


Year 4: Add and subtract fractions with the same denominator including bridging over whole numbers e.g. $\frac{7}{9} + \frac{4}{9} = \frac{11}{9}$ or $1\frac{2}{9}$	To add and subtract fractions, the denominators must be the same.	As before, use cubes and numicon to create the fractions:  $\frac{11}{6} - \frac{\square}{6} = \frac{\square}{6}$	Count up and down in fraction amounts on a number line.  *Twinkl* Count up in fraction amounts using paper cards. *I See*  	Make sure the numerators are the same, then add the denominators. If your answer is an improper fraction, convert it to a mixed number if the problem requires it. e.g. $\frac{7}{9} + \frac{4}{9} = \frac{11}{9}$ or $1\frac{2}{9}$

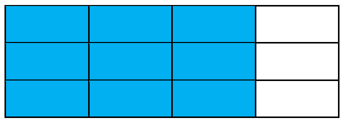
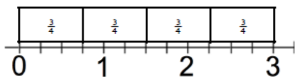
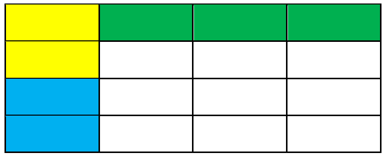
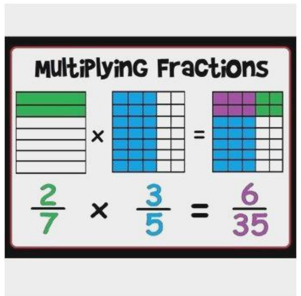
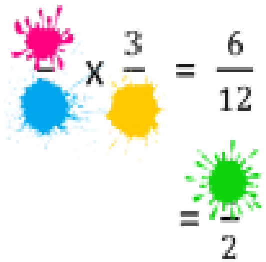
Year 5: Add and subtract fractions with different denominators where one denominator is a multiple of the another within the calculation	To add and subtract fractions, the denominators must be the same. Is there an easy way to find an equivalent fraction for just one fraction that makes the denominator the same? Refer to appendix for methods 1 and 2 for adding and subtracting fractions.	$\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} =$  Using paper strips, create fractions where one denominator is a multiple of the other and demonstrate how to convert one denominator into the other by splitting the fraction, so the denominators are equal.	$\frac{1}{3} - \frac{1}{12} =$  $\frac{1}{3} + \frac{5}{6} =$  	$\frac{2}{5} - \frac{1}{10} = \frac{4}{10} - \frac{1}{10} = \frac{3}{10}$

Year 6: Add and subtract fractions with different denominators	To add and subtract fractions, the denominators must be the same. Find common multiples of the denominators. Refer to appendix for methods 1 and 2 for adding and subtracting fractions.	Once they have combined their previous knowledge of +/- fractions with their knowledge of equivalent fractions, the process is as above.	 $\frac{1}{2}+\frac{1}{8}=\frac{4}{8}+\frac{1}{8}=\frac{5}{8}$  $\frac{1}{4}+\frac{3}{8}=\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$  	$\frac{3}{4}+\frac{2}{5}=\frac{15}{20}+\frac{8}{20}=\frac{23}{20}$ or $1\frac{3}{20}$ $\frac{3}{4}+\frac{2}{5}=\frac{15}{20}+\frac{8}{20}=\frac{23}{20}$ or $1\frac{3}{20}$

Year 6: Add and subtract fractions, including mixed numbers and improper fractions where the no bridging is required	As previous teaching points in other years. Refer to appendix for methods 1 and 2 for adding and subtracting fractions.	**Addition** Provide pupils with strips split into sevenths. Elicit that $\frac{15}{7}$ is the same $2\frac{1}{7}$  Elicit that $\frac{11}{7}$ is the same $1\frac{4}{7}$  Question: How many sevenths do we have altogether? Twenty six sevenths. Reminder: when adding/subtracting two fractions, the denominator stays the same – even when it is an improper fraction. Elicit that $\frac{26}{7}$ is equivalent to $3\frac{5}{7}$ Use a similar example to model the same for subtraction.	Oscar was running a race. He ran $3\,{}^{5}/_{8}$ of a kilometre in the first 15 minutes. He ran $2\,{}^{1}/_{4}$ of a kilometre in the second 15 minutes. How far had he run in total after 30 minutes? Share the following image:  Elicit that one full sheet of paper represents 1km. Agree that he has run five full kilometres because there are five full sheets. Ask: Oscar ran further than 5 km. How much further? Agree that Oscar has also ran $\frac{5}{8}$ km and $\frac{1}{4}\ km$. Establish that these are related fractions because one denominator is a multiple of the other. Share the following then get pupils to discuss what is happening: $3\frac{5}{8} + 2\frac{1}{4} = 3\frac{5}{8} + 2\frac{2}{8}$ $= (3 + \frac{5}{8}) + (2 + \frac{2}{8})$ $= (3 + 2) + (\frac{5}{8} + \frac{2}{8})$ $= (5 + \frac{7}{8})$ $= 5\frac{7}{8}$ *Collins Shanghai Y6 Unit 6.7* Use a similar example to model the same for subtraction.	$\mathbf{3\frac{3}{4} + \frac{1}{5} = 3\frac{19}{20}}$ $\mathbf{11\frac{3}{4} - 7\frac{1}{6} = 4\frac{7}{12}}$

Year 6: Add and subtract fractions, including mixed numbers and improper fractions where bridging is required	Refer to appendix for methods 1 and 2 for adding and subtracting fractions.	**Addition** Example: $3\frac{5}{7} + 2\frac{4}{7}$ Use the same method as previously with the strips, but get the children to cut up the fraction elements into the component sevenths. This would leave 5 wholes (5 strips of $\frac{7}{7}$) and nine $\frac{1}{7}$ parts. Elicit that nine $\frac{1}{7}$ parts is the same as $\frac{9}{7}$ or $1\frac{2}{7}$ parts. Combine 5 with $1\frac{2}{7}$ to get the answer of $6\frac{2}{7}$ **Subtraction** Example: $4\frac{2}{7} - 2\frac{5}{7}$ Use the same method as previously with the strips, but get the children to cut up the fraction elements into the component sevenths. This would leave 2 wholes (2 strips of $\frac{7}{7}$) but then they would realise that they cannot take $\frac{5}{7}$ away from $\frac{2}{7}$. Discuss what could be done next. Elicit that they could cut one of their two wholes into $\frac{7}{7}$. Going back to their original $4\frac{2}{7}$ they would the re-partition that into $3\frac{9}{7}$. With $3\frac{9}{7}$ in strips, they could remove the 2 wholes then the $\frac{4}{7}$ leaving them with $1\frac{5}{7}$.	  Calculate $3\frac{1}{4} - 1\frac{3}{4}$ $3\frac{1}{4}$ can become $2\frac{5}{4}$ 	For example $3\frac{3}{4} + \frac{7}{5} = 5\frac{3}{20}$ $11\frac{3}{4} - 7\frac{5}{6} = 3\frac{11}{12}$

Using resources effectively to teach…. X AND ÷ FRACTIONS

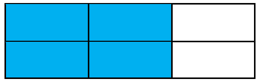
Obj	Teaching points	Concrete	Pictorial	Abstract
Year 6: Multiply simple pairs of proper fractions, writing the answer in its simplest form Multiply fractions by whole numbers.	When multiplying fractions by a whole integer, children will need to understand ***why*** it is only the numerator that changes – the denominator remains the same Explain the mistake: $\frac{3}{4} \times 5 = \frac{15}{20}$ Build in opportunities to discuss misconceptions and mistakes that could occur.	 Use Cuisenaire Rods and fraction towers to demonstrate multiplying fractions by an integer.  Using fraction towers to demonstrate the abstract: '3 lots of ¼ =' 2 lots of $^{2}/_{6}$ = Use bar models to work out 3 x ¾ =  Use a number line to work it out: 	Use a diagram to represent multiplying fractions. Build an array (as used when multiplying whole numbers For example ¼ x 2/4  Draw a bar and shade ¼ Draw an adjoining column and shade 2/4 . The shaded cells represent the total.(2/16 or 1/8) 	Solve: $\frac{2}{\_} \times \_ = \frac{6}{20} = \_$ $\_ \times \frac{\_}{5} = \frac{10}{60} = \_$ How many ways can you answer the following?  In each number sentence, replace the boxes with different whole numbers less than 20 so that the number sentence is true. Cont…

Year 6: Multiply simple pairs of proper fractions, writing the answer in its simplest form

Multiply fractions by whole numbers.

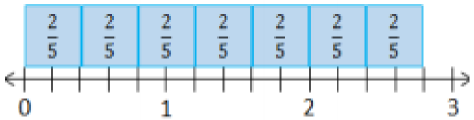
Draw your own bar model to work out the following:

2 x 2/3 =   4 x 2/3 =

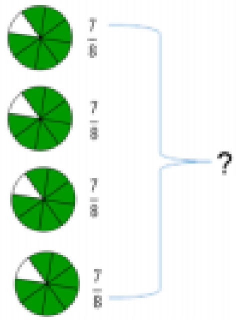


Find 2/4 of ¼ (2/4 x ¼)

$\frac{2}{5} \times 7$



$4 \times \frac{7}{8}$



...cont.

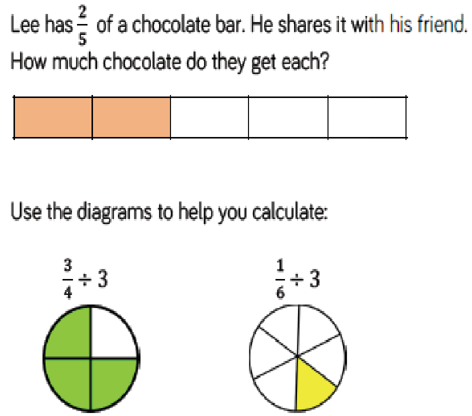
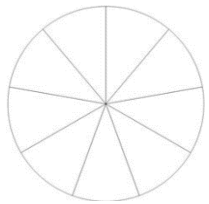
$$\frac{1}{\square} \times \frac{3}{\square} = \frac{\square}{\square}$$

$$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{8}{15}$$

$$\frac{2}{\square} \times \frac{5}{\square} < \frac{10}{\square}$$

$$\frac{\square}{\square} \div 3 = \frac{1}{\square}$$

$$\frac{\square}{\square} \div 3 > \frac{1}{4}$$

Year 6: Divide proper fraction by whole numbers.	Children need to know what happens to the denominator when it is divided by an integer and most importantly, ***why***. $\frac{3}{4} \div 2 = \frac{3}{8}$ Why have the quarters become eighths?	Using paper strips is a good way to demonstrate what happens to a fraction when it is divided by a whole integer i.e. 2 Fold the paper in half: ***What happens when I divide $^1/_2$ by 2?*** By folding each half into two parts, it becomes clear that quarters have been formed.	 Calculate the following and use the diagram to help you. $\frac{1}{8} \div 4 =$ 	Solve: $6 \div \frac{}{} = 9$ $\\_\\_\\_ \div \frac{2}{5} = 10$ $\frac{}{} \div \\_\\_\\_ = 5\frac{5}{6}$ Harjoht's Mum ordered pizza for the whole family. Harjoht ate ¼ of the pizza. His Mum, brother and sister ate the rest of the pizza. What fraction of the pizza did they get each? Molly's Mum ordered pizza for the whole family. Molly ate ¼ of the pizza. Six members of the family eat the remaining pizza. What fraction of the pizza did they get each?

Appendix 1

Adding and subtracting fractions

Method 1

Method 1 keeps the whole numbers and fraction parts separate largely, which is often conceptually easier for children. However, the exchange of one whole for an equivalent fraction when bridging is required in subtraction can prove difficult.

Adding Fractions Method 1

1. **Add whole number parts.**
2. **Make sure the denominators are the same if they are not already.**
3. **Add the numerators.**
4. **Simplify if possible.**
5. **Add up the whole number part and the fraction part if we need to.**

Example 1: 2/5 + 4/5

1. No whole numbers to include.
2. Denominators are already the same.
3. Add the numerators. 2/5 + 4/5 = 6/5.
4. 6/5 simplifies to 1 1/5.
5. No whole number parts to add up, so **1 1/5** is our answer.

Example 2: 9 2/3 + 6 4/5

1. Add up the whole numbers: 9 + 6 = 15 (keep that for later).
2. Make the denominators the same. Thirds and fifths can both be converted to fifteenths. 2/3 = 10/15 (multiplying numerator and denominator by 5 to keep the fraction equivalent). 4/5 = 12/15 (multiplying numerator and denominator by 3 to keep the fraction equivalent).
3. Add the numerators. 10/15 + 12/15 = 22/15
4. There are 15/15 in one whole, so we can simplify this to 1 7/15.

5. Add the two parts together: 15 + 1 7/15 = **16 7/15**

Example 3: 3 3/4 + 7/5

1. 7/5 is an improper fraction; we convert it to 1 2/5. Add up the whole numbers: 3 + 1= 4 (keep that for later).
2. Make the denominators the same. Quarters and fifths can both be converted to twentieths. 3/4 = 15/20 (multiplying numerator and denominator by 5 to keep the fraction equivalent). 2/5 = 8/20 (multiplying numerator and denominator by 4 to keep the fraction equivalent).
3. Add the numerators. 15/20 + 8/20 = 23/20
4. There are 20/20 in one whole, so we can simplify this to 1 3/20.
5. Add the two parts together: 4 + 1 3/20 = **5 3/20**

Subtracting Fractions Method 1a

1. **Subtract whole number parts.**
2. **Make sure the denominators are the same if they are not already.**
3. **If the left-side fraction part is smaller than the right-side fraction part, we need to exchange one whole from our whole number part to make the right-side fraction part big enough.**
4. **Subtract the numerators.**
5. **Simplify if possible.**
6. **Add up the whole number part and the fraction part if we need to.**

Example 1: 7/8 - 1/4

1. No whole number parts to subtract.
2. Make the denominators the same. Quarters can be converted to eighths. 1/4 = 2/8 (multiplying numerator and denominator by 2 to keep the fraction equivalent). We now have 7/8 – 2/8.
3. The left-side fraction is big enough.
4. Subtract the numerators. 7/8 – 2/8 = 5/8.
5. 5/8 does not simplify.
6. There is no whole number part here. Our answer is **5/8**.

Example 2: 11 3/4 – 7 5/6

1. Subtract whole number parts. 11 – 7 = 4 (keep that for later).
2. Make the denominators the same. Quarters and sixths can be converted to twelfths. 3/4 = 9/12 (multiplying numerator and denominator by 3 to keep the fraction equivalent). 5/6 = 10/12 (multiplying numerator and denominator by 2 to keep the fraction equivalent). We now have 9/12 – 10/12.
3. The left-side fraction is smaller than the right-side fraction, so we need to make it bigger. We do this by exchanging one from our whole number part for 12/12. Our whole number part is now 3. By adding 12/12 to 9/12, our fraction calculation is now 21/12 – 10/12.
4. Subtract the numerators. 21/12 – 10/12 = 11/12.
5. 11/12 does not simplify.
6. Add together the whole number and fraction parts. 3 + 11/12 = **3 11/12**.

<u>Subtracting Fractions Method 1b</u>

1. **Make sure the denominators are the same if they are not already.**
2. **If the left-side fraction part is smaller than the right-side fraction part, we need to exchange one whole from our whole number part to make the right-side fraction part big enough.**
3. **Subtract whole number parts.**
4. **Subtract the numerators.**
5. **Simplify if possible.**
6. **Add up the whole number part and the fraction part if we need to.**

<u>Example 1: 7/8 - 1/4</u>

1. Make the denominators the same. Quarters can be converted to eighths. 1/4 = 2/8 (multiplying numerator and denominator by 2 to keep the fraction equivalent). We now have 7/8 – 2/8.
2. The left-side fraction is big enough.
3. There are no whole number parts to subtract.
4. Subtract the numerators. 7/8 – 2/8 = 5/8.
5. 5/8 does not simplify.
6. There is no whole number part here. Our answer is **5/8**.

<u>Example 2: 11 3/4 – 7 5/6</u>

1. Make the denominators the same. Quarters and sixths can be converted to twelfths. 3/4 = 9/12 (multiplying numerator and denominator by 3 to keep the fraction equivalent). 5/6 = 10/12 (multiplying numerator and denominator by 2 to keep the fraction equivalent). We now have 9/12 – 10/12.
2. The left-side fraction is smaller than the right-side fraction, so we need to make it bigger. We do this by exchanging one from our whole number part for 12/12. Our whole number part on the left is 11, so that now becomes 10. By adding 12/12 to 9/12, our fraction calculation is now 21/12 – 10/12.
3. This makes our new calculation **10 21/12 – 7 10/12.** Subtract whole number parts. 10 – 7 = 3.
4. Subtract the numerators. 21/12 – 10/12 = 11/12.
5. 11/12 does not simplify.
6. Add together the whole number and fraction parts. 3 + 11/12 = **3 11/12**.

Method 2

Method 2 converts mixed numbers to improper fractions. This simplifies trickier calculations involving bridging. However, arithmetic errors are more likely to occur using this method, and this risk grows in proportion with the value of the numbers involved.

Adding Fractions Method 2

1. **Make sure the denominators are the same if they are not already.**
2. **If they are mixed numbers, convert them to improper fractions.**
3. **Add the numerators.**
4. **Convert any improper fractions back to mixed numbers if required.**
5. **Simplify if possible.**

Example 1: 2/5 + 4/5

1. Denominators are already the same.
2. Proper fractions with no whole numbers, so no need to convert.
3. Add the numerators. 2/5 + 4/5 = 6/5.
4. 6/5 converts to 1 1/5.
5. Nothing to simplify. Answer is **1 1/15**.

Example 2: 9 2/3 + 6 4/5

1. Make the denominators the same. Thirds and fifths can both be converted to fifteenths. 2/3 = 10/15 (multiplying numerator and denominator by 5 to keep the fraction equivalent). 4/5 = 12/15 (multiplying numerator and denominator by 3 to keep the fraction equivalent).
2. They are mixed numbers. Convert 9 10/15 to 145/15 and 6 12/15 to 102/15.
3. Add the numerators. 145/15 + 102/15 = 247/15.
4. 247/15 converts to 16 7/15.
5. Nothing to simplify. Answer is **16 7/15**.

Subtracting Fractions Method 2

1. **Make sure the denominators are the same if they are not already.**
2. **If they are mixed numbers, convert them to improper fractions.**
3. **Do left numerators subtract right numerator.**
4. **Convert any improper fractions back to mixed numbers if required.**
5. **Simplify if possible.**

Example 1: 7/8 - 1/4

1. Make the denominators the same. Quarters can be turned into eighths. 1/4 = 2/8 (multiplying numerator and denominator by 2 to keep the fraction equivalent).
2. Proper fractions with no whole numbers, so no need to convert.
3. Subtract the numerators. 7/8 - 2/8 = 5/8.
4. 5/8 is a proper fraction, so nothing to convert.
5. Nothing to simplify. Answer is **5/8**.

Example 2: 11 3/4 – 7 5/6

1. Make the denominators the same. Quarters and sixths can both be converted to twelfths. 3/4 = 9/12 (multiplying numerator and denominator by 3 to keep the fraction equivalent). 5/6 = 10/12 (multiplying numerator and denominator by 2 to keep the fraction equivalent).
2. They are mixed numbers. Convert 11 9/12 to 141/12 and 7 10/12 to 94/12.
3. Subtract the numerators. 141/12 - 94/12= 147/12.
4. 147/12 converts to 12 3/12.
5. 3/12 simplifies to 1/4 (dividing numerator and denominator by 3 to keep the fraction equivalent). Answer is **12 1/4**.